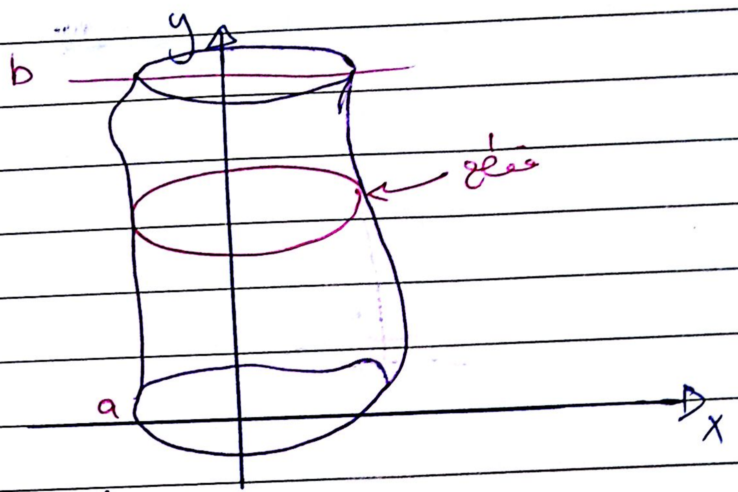
Volumes:

Given a solid



$$V = \int_a^b A(y)\,dy$$

A(y) equals the area cross section perpendicular to the y-axis of levels.

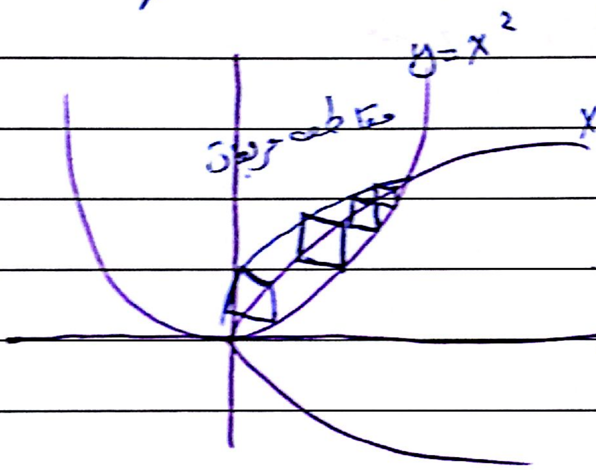
$$V = \int_c^d A(x)\,dx$$

A(x) is the area of this section of the solid perpendicular to the x-axis at level x

Ex

Find the volume solid where base in the region bounded by the curves $y = x^3$ and $x = y^2$ such that each section perpendecular to the x-axis is:

① a square
② a semi circles
③ an equilatral triangle.



$x = y^2 \longrightarrow y = \sqrt{x}$

نأخذ مقطع على شكل مربع

طول الضلع العلوي - السفلي

مساحة المقطع $A(x) = (\sqrt{x} - x^2)^2$

$$V = \int_0^1 (\sqrt{x} - x^2)^2 \, dx$$

$$= \int_0^1 x - 2x^2\sqrt{x} + x^4 \, dx$$

$$\vdots$$

$$= \frac{9}{70} \text{ unit}$$

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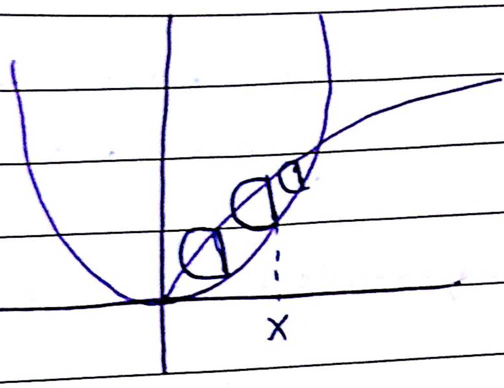
$\boxed{2}$



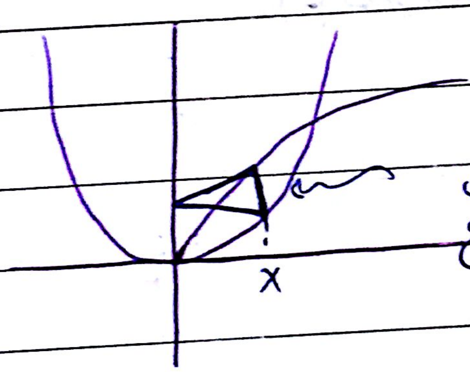
قطر الدائرة

$= \sqrt{x} - x^2$

$$Area = \pi \left(\frac{\sqrt{x} - x^2}{2}\right)^2 * \frac{1}{2}$$ لأنها نصف دائرة

$$V = \int_0^1 \frac{\pi}{8} \left(\sqrt{x} - x^2\right)^2$$

$\boxed{3}$



$$A = \frac{\sqrt{3}}{4} \left(\sqrt{x} - x^2\right)^2$$

$$V = \int_0^1 \frac{\sqrt{3}}{4} \left(\sqrt{x} - x^2\right)^2$$

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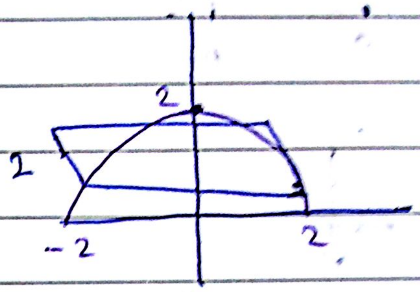
* Find the volume solid where base in the region bounded by the curves $y = \sqrt{4-x^2}$ and x-axis

such that each section perpindecular to the y-axis is arectangle where hight = 2 units

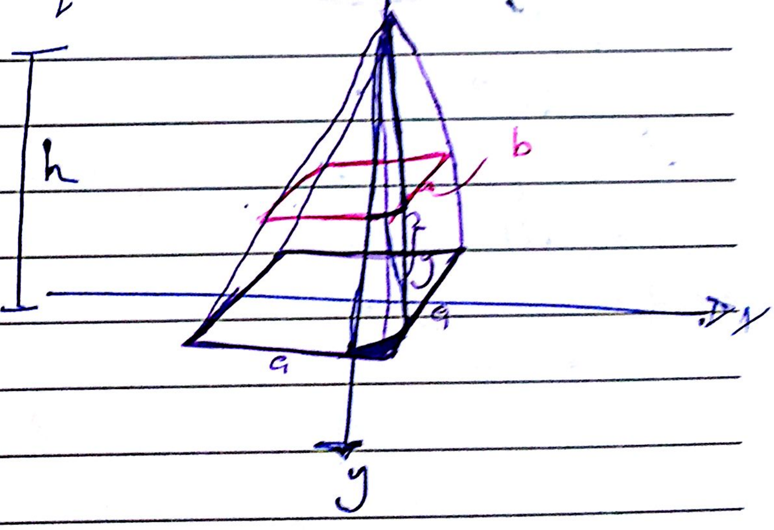
$$A(y) = 2x(2)$$
$$= 4x$$
$$= 4\sqrt{4-y^2}$$



$$V = \int_0^2 4\sqrt{4-y^2} = \pi \text{ unit}^3$$

$y^2 = 4 - x^2$
$x = \mp\sqrt{4-y^2}$

Ex Find the volume of the right pyramid where hight is h and square base with side lenghth = a



[illegible]

$$\frac{\frac{b}{2}}{\frac{a}{2}} = \frac{h-y}{h}$$

$$b = a\left(1 - \frac{y}{h}\right)$$

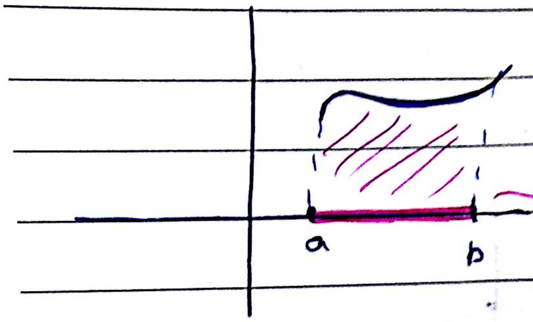
$$A(y) = a^2\left(1 - \frac{y}{h}\right)^2$$

$$V = \int_0^H a^2\left(1 - \frac{y}{h}\right)^2 dy$$

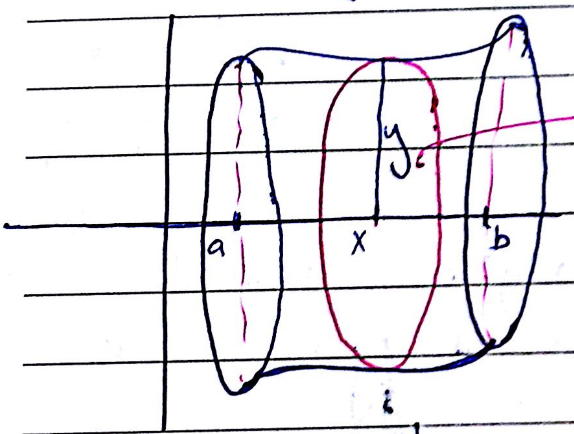
$$= -h a^2 \frac{\left(1 - \frac{y}{h}\right)^3}{3} \Bigg]_0^h$$

$$= 0 + ha^2 \frac{\left(1 - \frac{0}{h}\right)^3}{3} = \frac{1}{3} ha^2$$

Volumes by solid revolution



لو اعتبرنا منطقة معرفة ومثبتة على ab وتدور عليه (يعني ab هو محور دوران)



نصف القطر y

$y = f(x)$

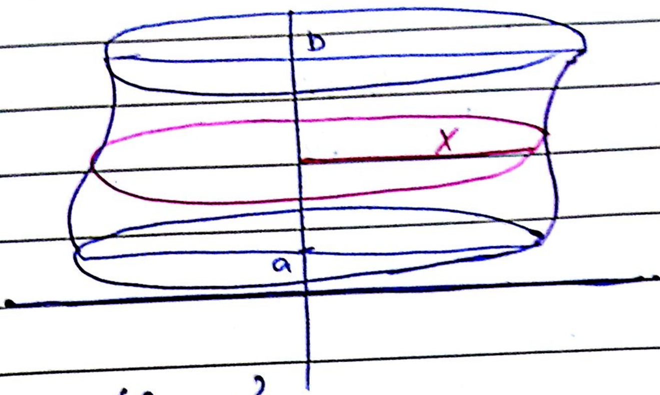
نجد حجم الاسطوانة لو نجزئ المقطع على x-axis's وتكون دائماً المقاطع دوائر ومساحة الدائرة = π نق^2 :

$$A(x) = \pi (f(x))^2$$

نجد الحجم

$$V = \int_a^b \pi (f(x))^2 \, dx$$

#

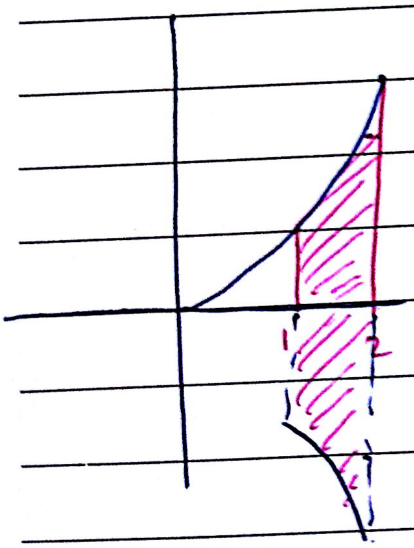


$$A(y) = \pi (f(y))^2$$

$$\text{volume} = \int_a^b \pi (f(y))^2 \, dy$$

Ex Find the volume of the solid generated by rotating the region bounded by

$y = x^2$, x-axis, $x = 1$ and $x = 2$
about the x-axis



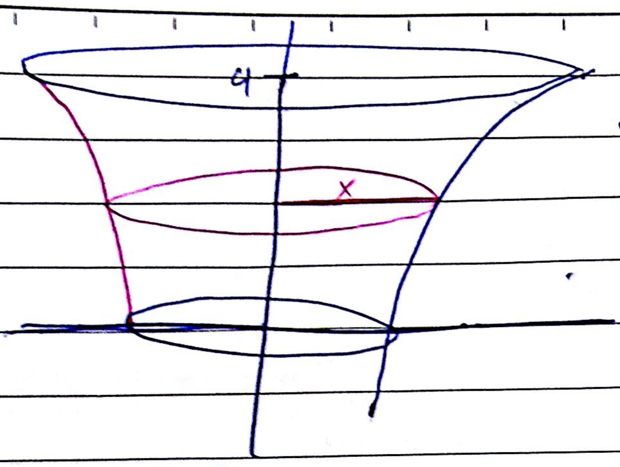
$$V = \int_1^2 \pi (x^2)^2 \, dx$$

$$= \pi \frac{x^5}{5} \Big]_1^2$$

$$= \pi \frac{(2)^5}{5} - \pi \frac{1}{5}$$

$$= \frac{31\pi}{5} \text{ unit}^3$$

Ex



$y = \ln(x)$

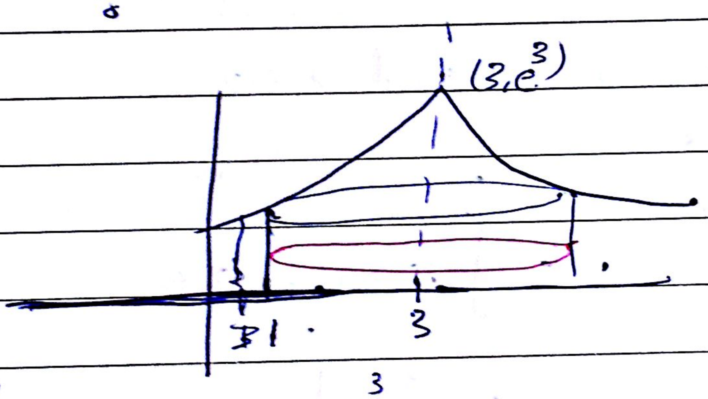
$x = e^y$

$A(y) = \pi (e^y)^2$

$$V = \int_0^4 \pi e^{2y}$$

$$\frac{\pi}{2} e^{2y} \Big]_0^4 = \frac{\pi}{2} e^8 - \frac{\pi}{2}$$

Ex



ab at $x = 3$

الحجم بالجزء السفلي يختلف عن حجم بالجزء العلوي

السفلي هو عبارة عن حجم لاسطوانة

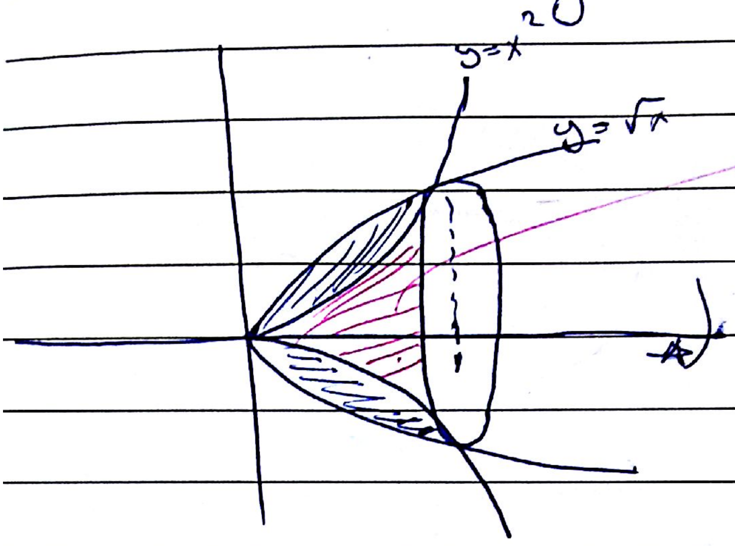
$$V = \int_0^e \pi (4)\, dy + \int_e^{e^3} \pi \underbrace{(3 - \ln(y))^2}_{\text{بالاجزاء}} dy$$

نكمل كل

*كل ما تغير الاقتران

نعمل التكامل

Volume by washers



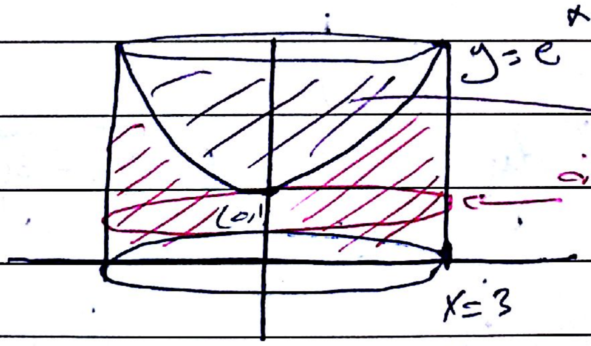
بيكون داخله تجويف

بيتطلعلها عنه

نشوفها حول محور السينات

$$V=\int_0^1 \pi(\sqrt{x})^2\,dx-\int_0^1 \pi(x^2)^2\,dx$$

$$=\pi\frac{x^2}{2}\Big]_0^1-\pi\frac{x^5}{5}\Big]_0^1$$

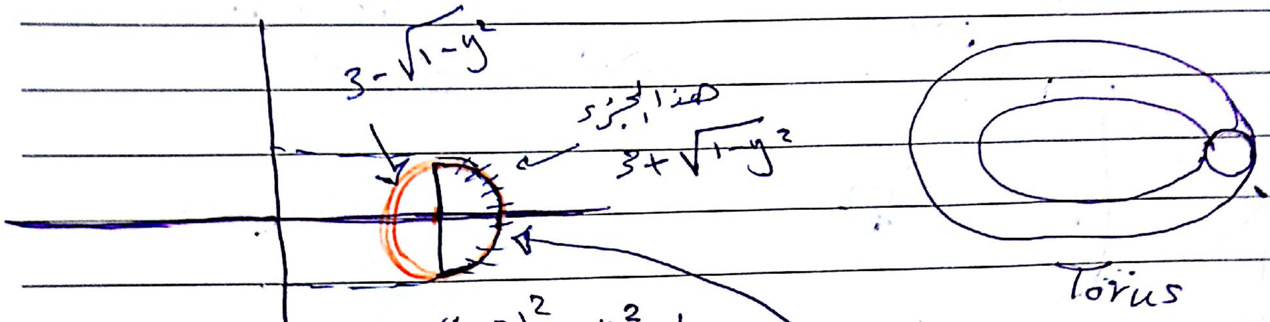


تجويف

حجم اسطوانة

$$A(y)=\int_0^1 \pi(3)^2\,dy+\int_1^{e^x}\pi(3)^2\,dy-\int_1^{e^x}\pi(\ln(y))^2\,dy$$

A Find the volume of the solid generated by rotating the region inside the circle about the y-axis.



$(x-3)^2+y^2=1$

نكتب x بدلالة y

الحجم الناتج من دوران هذا الجزء

$x = 3 \mp \sqrt{1-y^2}$

حجم الجزء الخارجي

حجم الاسطوانة التجويف الداخلي

$$V = \int_{-1}^{1} \pi\left(3+\sqrt{1-y^2}\right)^2 - \int_{-1}^{1} \pi(3)^2\,dy$$

حجم الجزء الداخلي

$$+ \int_{-1}^{1} \pi(3)^2\,dy - \int_{-1}^{1} \pi\left(3-\sqrt{1-y^2}\right)^2 dy$$

$$= \int_{-1}^{1} \pi\left(3+\sqrt{1-y^2}\right)^2 - \int_{-1}^{1} \pi\left(3-\sqrt{1-y^2}\right)^2 dy$$

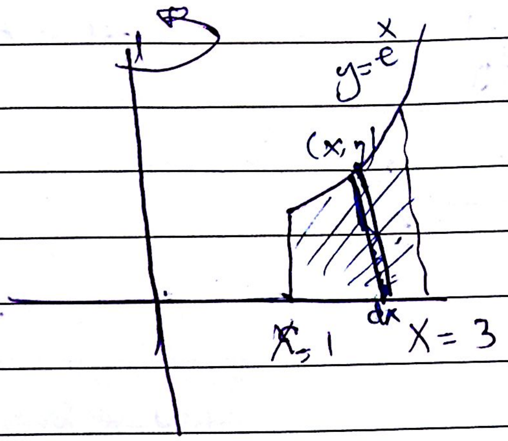
فرق بين مربعين

$$= \int_{-1}^{1} \pi[6]\left[2\sqrt{1-y^2}\right] dy$$

$$= 12\int_{-1}^{1} \sqrt{1-y^2}\,dy \longrightarrow \text{مساحة نصف الدائرة}$$

$= 6\pi^2$ unit^3

volume by Cylindrical shells. (shell method)

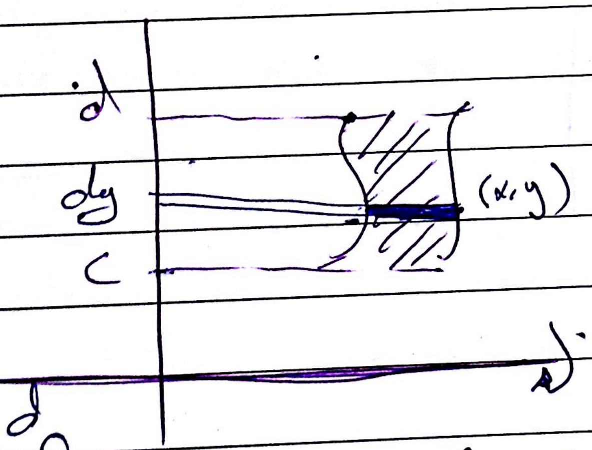


$dV = 2\pi$ (radius) (height) dx

$V = \int 2\pi$ (radius) (hight) dx

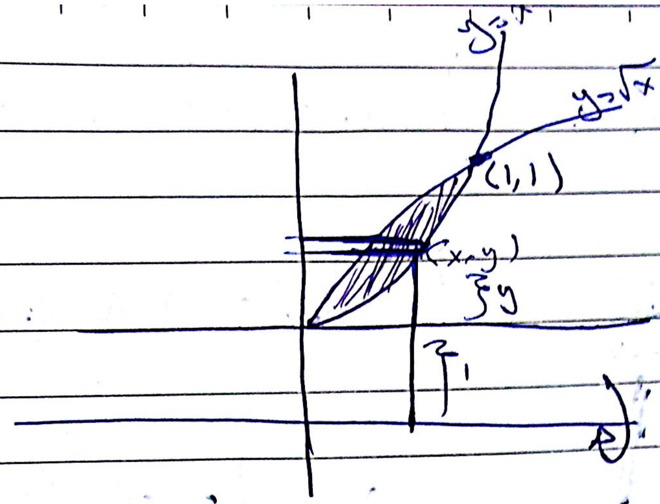
if the rotation is about the y-axis or line: is parallel to y-axis

$$V = \int_1^3 2\pi (x)(e^x)\, dx$$



$$V = \int_c^d 2\pi \text{ (radius) (hight) } dy$$

* Ex



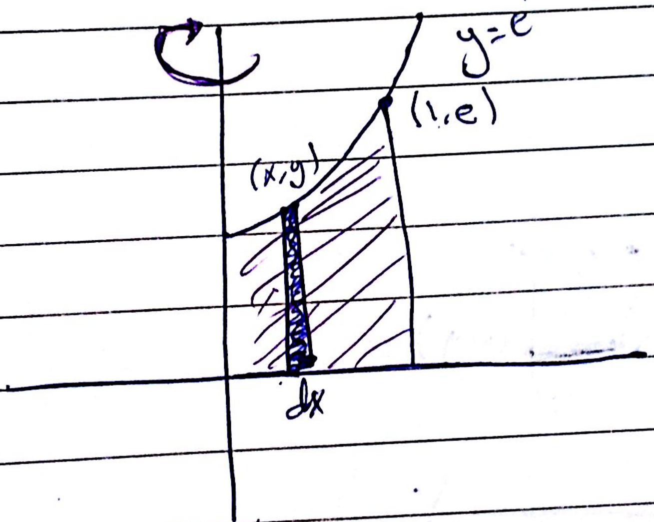
about $y=-1$

① نأخذ شريحة موازية لمحور الدوران

ارتفاع الشريحة

$$V=\int_0^1 2\pi\,(y+1)\,(\sqrt{y}-y^2)\,dy$$

Ex



$$V=\int_0^1 2\pi(x)\,(e^{x^2})\,dx$$

$$=\pi e^{x^2}\Big]_0^1$$

$$=\pi e-\pi=\pi(e-1)\ \text{unit}^3$$

* $\int_0^1 \sqrt[7]{1-x^3}\, dx - \int_0^1 \sqrt[3]{1-x^7}\, dx$

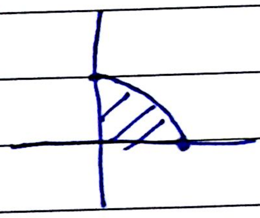
Consider

$\int_0^1 \sqrt[7]{1-x^3}$ $\qquad f(x) = \sqrt[7]{1-x^3}$

$f(0) = 1$

$f(1) = 0$

= The area of the region bounded by the curve $y = \sqrt[7]{1-x^3}$ and x-axis and $x=0$, $x=1$



We want to find the area of the same region integrating over y

$y = \sqrt[7]{1-x^3} \rightarrow y^7 = 1-x^3 \rightarrow x^3 = 1-y^7$

$x = \sqrt[3]{1-y^7}$

Area $= \int_0^1 \sqrt[3]{1-y^7}$

but $\int_0^1 \sqrt[3]{1-x^7}\, dx = \int_0^1 \sqrt[3]{1-y^7}\, dy$

So $\int_0^1 \sqrt[7]{1-x^3} = \int_0^1 \sqrt[3]{1-x^7}\, dx$

$= \int_0^1 \sqrt[7]{1-x^3}\, dx - \int_0^1 \sqrt[3]{1-x^7}\, dx = 0$ //

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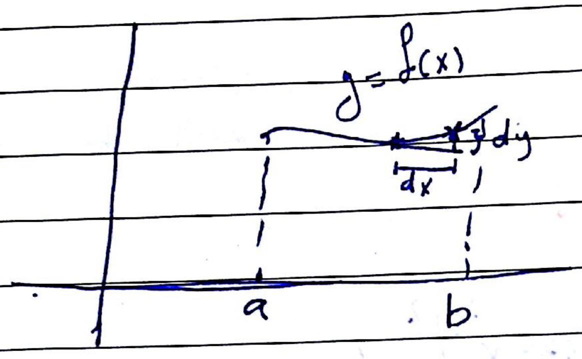
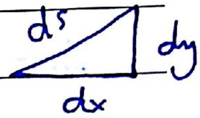
CH8
Arc. lenghth

*نأخذ جزء صغير من المنحنى لأنه صغير جداً
نستطيع اعتباره خط مستقيم





$$(ds)^2 = (dx)^2 + (dy)^2$$

$$ds = \sqrt{(dx)^2 + (dy)^2}$$

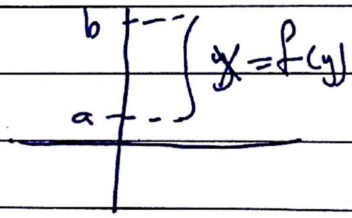
lenght of the curve =

$$ds = \frac{\sqrt{(dx)^2 + (dy)^2}}{dx} \cdot dx$$

$$= \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$S = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$ ← لو بنكامل على X

*



$$ds = \frac{\sqrt{(dx)^2 + (dy)^2}}{dy} \cdot dy$$

$$ds = \sqrt{\left(\frac{dx}{dy}\right)^2 + 1}\; dy$$

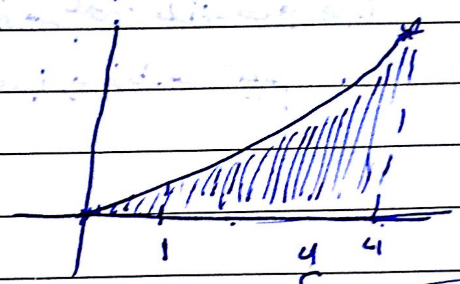
lenght of the curve $= S = \displaystyle\int_a^b \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

8

$\cancel{\int (3) x^2 \sqrt{x^5}} = \cancel{\frac{1}{3} \sqrt{x}}$

<u>Ex</u>

Find the lenght of the curve $y = x^{\frac{3}{2}}$, $0 \leq x \leq 4$.



Arc lenghth $= \int_0 \sqrt{1 + \left(\frac{3}{2} x^{\frac{1}{2}}\right)^2} \, dx$

$$= \int_0^4 \sqrt{1 + \frac{9}{4} x} \, dx$$

$$= \frac{4}{9} \left(1 + \frac{9}{4} x\right)^{\frac{3}{2}} \cdot \frac{2}{3} \Bigg]_0^4$$

$$= \frac{8}{27} \left(1 + \frac{9}{4} (4)\right)^{\frac{3}{2}} - \frac{8}{27} \left(1 + \frac{9}{4} (0)\right)^{\frac{3}{2}}$$

$=$

Ex $\frac{10}{55}$ $\Big|$ $y = \frac{x^3}{6} + \frac{1}{2x}$, $\frac{1}{2} \le x \le 1$

$$\frac{dy}{dx} = \frac{1}{2}x^2 - \frac{1}{2x^2}$$

leught of the Curve $= \displaystyle\int_{\frac{1}{2}}^{1} \sqrt{1 + \left(\frac{1}{2}x^2 - \frac{1}{2x^2}\right)^2}$

$$= \int_{\frac{1}{2}}^{1} \sqrt{1 + \left(\frac{1}{4}x^4 - \frac{1}{2} + \frac{1}{4x^4}\right)}$$

$$= \int_{\frac{1}{2}}^{1} \sqrt{1 + \frac{1}{4}x^4 - \frac{1}{2} + \frac{1}{4x^4}}$$

$$= \int_{\frac{1}{2}}^{1} \sqrt{\frac{1}{4}x^4 + \frac{1}{4x^4} + \frac{1}{2}} \longleftarrow \text{مربع كامل}$$

$$= \int_{\frac{1}{2}}^{1} \sqrt{\left(\frac{1}{2}x^2 + \frac{1}{2x^2}\right)^2}$$

$$= \int_{\frac{1}{2}}^{1} \frac{1}{2}x^2 + \frac{1}{2x^2}$$

$$= \frac{1}{6}x^3 - \frac{1}{2x} \Big]_{\frac{1}{2}}^{1}$$

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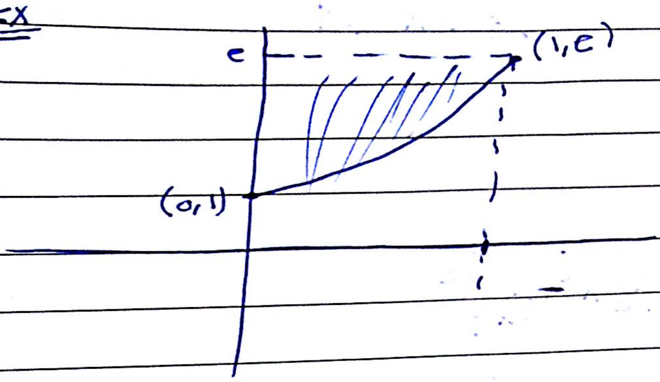
Ex



*نستطيع ان نكامل على X أو y

$X = \ln(y)$

$1 \le y \le e$

lenght of the curve $= \int_1^e \sqrt{1+\left(\frac{1}{y}\right)^2}\, dy$

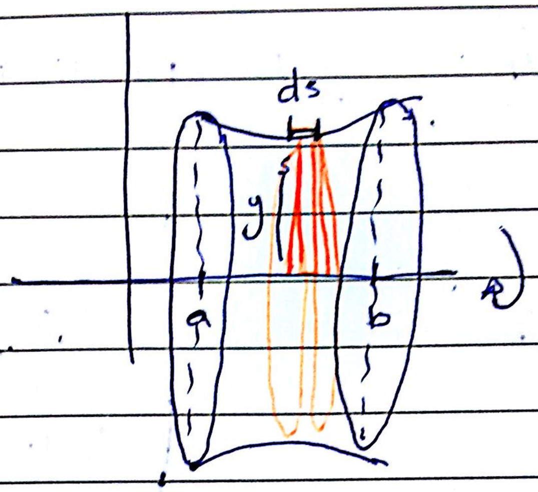
$$= \int_1^e \frac{\sqrt{y^2+1}}{y}\, dy$$ → trigonemetric subsitution.

$y = \tan\theta \qquad dy = \sec^2\theta\, d\theta$

$$= \int \frac{\sec\theta \cdot \sec^2\theta\, d\theta}{\tan\theta}$$

~~II~~ Area of surface revolution

Given $y = f(x)$ $\quad a \le x \le b$



Find the area of the surface obtained by rotating the Curve about X-axis

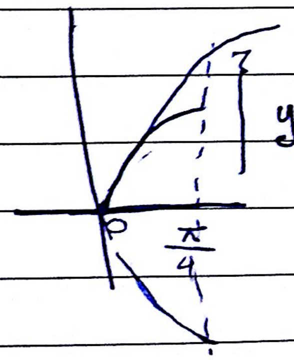
$$\int_a^b 2\pi (y)\, ds$$

$$= \int_a^b 2\pi \left(f(x)\right)\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

Ex Find the area of the surface obtained by rotating the Curve $y = \sin x$ $\quad 0 \le x \le \frac{\pi}{4}$ about ① the X-axis

② $y = -1$

① $$S = \int_0^{\frac{\pi}{4}} 2\pi \sin x \sqrt{1 + \cos^2 x}\, dx$$



$u = \cos x$

$du = -\sin x\, dx$

$$\cancel{\int 2\pi \sin x\, dx} = \int 2\pi - (1 + u^2)^{\frac{1}{2}}$$

$$\cancel{= 2\pi - (1+u} = \int -2\pi \sqrt{1 + u^2}$$ — تعويض مثلثي

$u = \tan\theta$

$du = \sec^2\theta\, d\theta$

$$= -2\pi \int \sec^3\theta\, d\theta$$

$$= -2\pi \left(\frac{1}{2}\sec\theta \tan\theta + \frac{1}{2}\ln\left|\sec\theta + \tan\theta\right|\right)$$

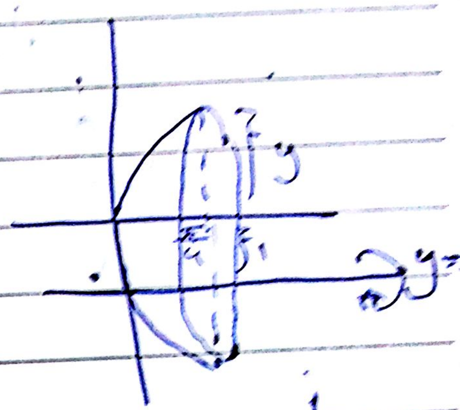
$= -2\pi \sqrt{1+u^2}\,(u) + \pi \ln\left|\sqrt{1+u^2}+u\right|$

$$= -\pi \left[\sqrt{1+\cos^2 x}\,.\cos x + \pi \ln\left|\sqrt{1+\cos^2 x}+\cos x\right|\right]_0^{\frac{\pi}{4}}$$

$$= -\pi \left[\sqrt{1+\frac{1}{2}} * \frac{1}{\sqrt{2}} + \pi \ln\left|\sqrt{1+\frac{1}{2}} + \frac{1}{\sqrt{2}}\right| - \sqrt{2}\,.(1) + \pi \ln\left|\sqrt{2}+1\right|\right.$$

$\boxed{2}$

$$S_{Area} = \int_0^{\frac{\pi}{4}} 2\pi(\sin x+1)\sqrt{1+\cos^2 x}\,dx$$



* Set up the ~~curve~~ integral that gives the area of the surface ~~rotating~~ abotained by rotating the curve

$y = \sin x \quad 0 \le x \le \frac{\pi}{4}$

about $y = -1$

اللي بيتناف
هو نوف نقص
الدوران

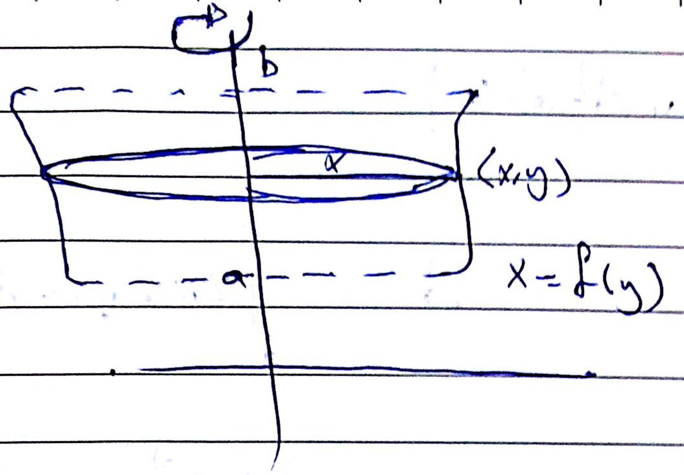
يعني فقط
اكتب التكامل
دون حل

*

Area of the surface =

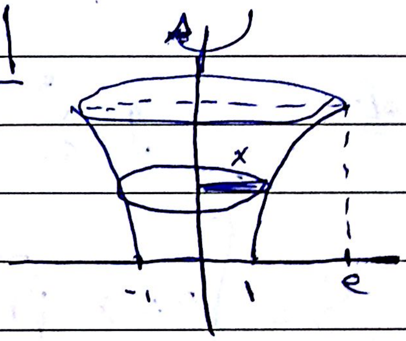
$$\int_{0}^{\frac{\pi}{4}} \cancel{2\pi(\sin x}$$

نفس الشيء
تكون كله y



Ex Find the area of the surface obtained by rotating $y=\ln(x)$, $0 \le x \le e$ about the y-axis

$\boxed{\text{method 1}}$



$$\text{Area} = \int_{1}^{e} 2\pi \underbrace{x}_{\text{نص قطر الدوران}} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_{1}^{e} 2\pi x \sqrt{1+\frac{1}{x^2}}\, dx \qquad = \int_{1}^{e} 2\pi \cancel{x} \frac{\sqrt{1+x^2}}{\cancel{x}}$$

$$= 2\pi \int_{1}^{e} \sqrt{1+x^2}$$

$x = \tan\theta$

$dx = \sec^2\theta\, d\theta$

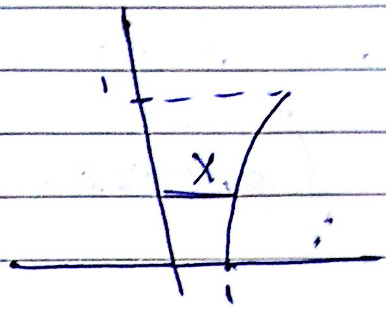
$$= 2\pi \int_{1}^{e} \sec^3\theta$$

$$= 2\pi\left[\frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln|\sec\theta+\tan\theta|\right]$$

نعوض

method 2)

$x = e^{y}$



$$A = \int_0^1 2\pi x \sqrt{1 + e^{2y}}$$

$$\int_0^1 2\pi e^{y} \sqrt{1 + e^{2y}} \, dy$$

$u = e^{y}$

$du = e^{y} dy$

$dy = \frac{du}{u}$

نكمل التكامل والتعويض ☺ ##

Ex $y = \frac{x^3}{6} + \frac{1}{2x}$, $1 \leq x \leq 2$ about y-axis

* هذا السؤال صعب لأنه تغير x بدلالة y

لذلك نستخدم الطريقة الأولى

$$S = \int_1^2 2\pi x \sqrt{1 + \left(\frac{x^2}{2} + \frac{-1}{2x^2}\right)^2} \, dx$$

انتبه o.o

مربع كامل

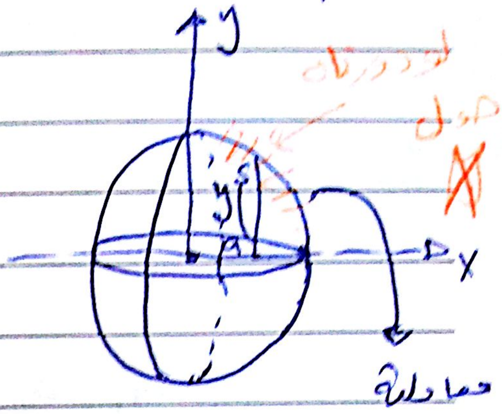
$$= \int_1^2 2\pi x \sqrt{1 + \frac{x^4}{4} - \frac{1}{2} + \frac{1}{4x^4}} \, dx$$

$$= \int_1^2 2\pi \sqrt{\left(\frac{x^2}{2} + \frac{1}{2x^2}\right)^2} \, dx$$

$$= \int_1^2 2\pi \left(\frac{x^3}{2} + \frac{1}{2x}\right) dx$$

$$= 2\pi \left[\frac{x^4}{8} + \frac{1}{2} \ln|x|\right]_1^2$$

Ex Find the area of the surface of the sphere of radius a.



$$S = \int_{-a}^{a} 2\pi y \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$y = \sqrt{a^2-x^2}$

$dy = \frac{1}{2}(a^2-x^2)^{-\frac{1}{2}} \cdot -2x$

$$= \int_{-a}^{a} 2\pi \sqrt{a^2-x^2} \sqrt{1+\left(\frac{1}{2}(a^2-x^2)^{-\frac{1}{2}} \cdot -2x\right)^2}$$

$$= \int_{-a}^{a} 2\pi \sqrt{a^2-x^2} \sqrt{1+\frac{x^2}{a^2-x^2}}$$

$$= \int_{-a}^{a} 2\pi \cancel{\sqrt{a^2-x^2}} \frac{a}{\cancel{\sqrt{a^2-x^2}}}$$

$$\therefore = \int_{-a}^{a} 2\pi a \, dx = 2\pi a x \Big]_{-a}^{a} = 4a^2\pi$$

المتتاليات والمتسلسلات

Sequences and Series. !!

1. Sequences: is an ordered set of numbers of the form

$a_1, a_2, a_3, a_4, \ldots$

where a_1, a_2, a_3 are numbers → called the
أعداد حقيقية

terms of the sequence

Ex

a_1: is Called the 1st term الحد الأول

a_2: " " " 2nd "

a_3: " " " 3rd "

⋮

الحد العام

a_n: " " " n-th term (the general term)

Ex

1) $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$

The first term $a_1 = 1$

⋮

The tenth term $= \frac{1}{10}$

$a_n = \frac{1}{n}$ #

سخيف !!

2) $1, -1, 1, -1, 1, -1, ---$

$a_5 = 1$

$a_{10} = -1$

ممكن نكتبه بطريقتين :- ✱

✱ $a_n = \begin{cases} 1 , & n \text{ is odd} \\ -1 , & n \text{ is even} \end{cases}$

✱ $a_n = (-1)^{n-1}$ or $a_n = (-1)^{n+1}$

3) $\frac{\ln(1)}{1}, \frac{\ln 2}{2}, \frac{\ln 3}{3}, ---$

$a_n = \frac{\ln(n)}{n}$ #

✱ In first example

$f(x) = \frac{1}{x}$

✱ A sequance can be viewed as a function who's domain is the set of the positive integers {1, 2, 3

$f(n) = a_n$

* A sequence $a_1, a_2, a_3, \ldots$ can be written simply as $\{a_n\}_{n=1}^{\infty}$.

قانون الحد العام — $n=1$ — تبدأ العداد

ممكن يبدأ من أي رقم (مو شرط 1)

Ex: $\left\{\frac{1}{n}\right\}_{n=1}^{\infty}$ is the sequence

تفسيرها — $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$

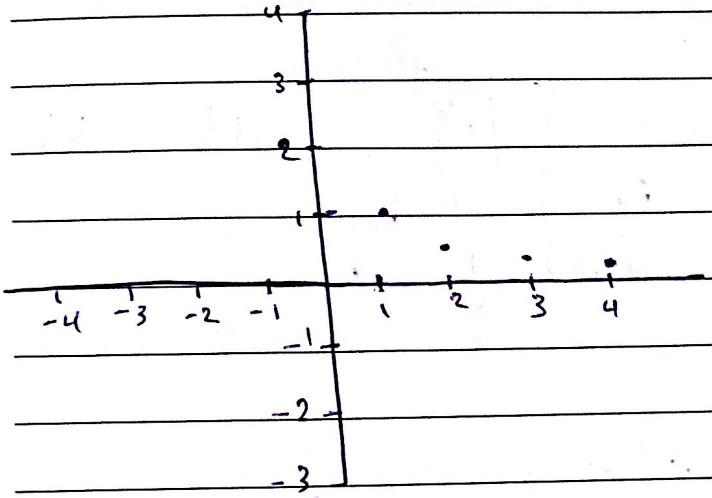
* Graph of sequence: رسم (بيان) المتتالية.

Given a sequence $\{a_n\}_{n=1}^{\infty}$ then the graph of this sequence is the set of all ordered pairs $\{(n, a_n) : n = 1, 2, 3, 4, \ldots\}$

Example :-

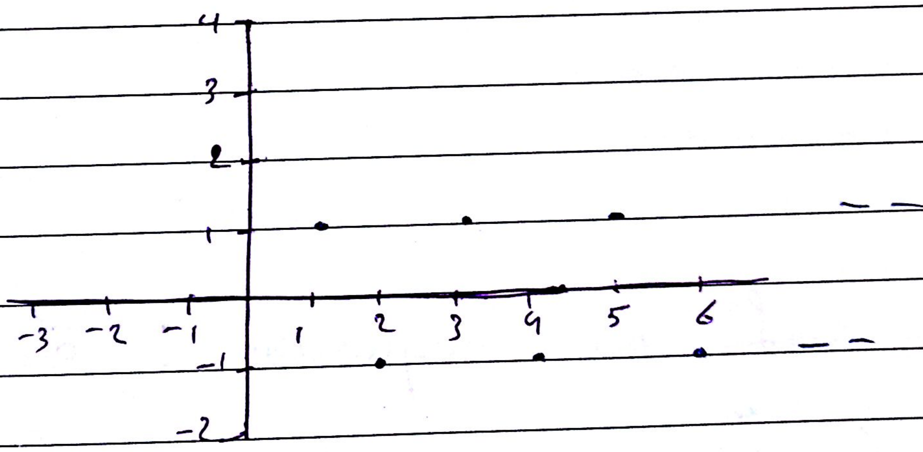
Draw the graph of the sequence :-

1. $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$



① نرسم الاحداثيات ونفرض القيم
(نرسم ونفرض نقاط)

2. $1, -1, 1, -1, 1, \ldots$

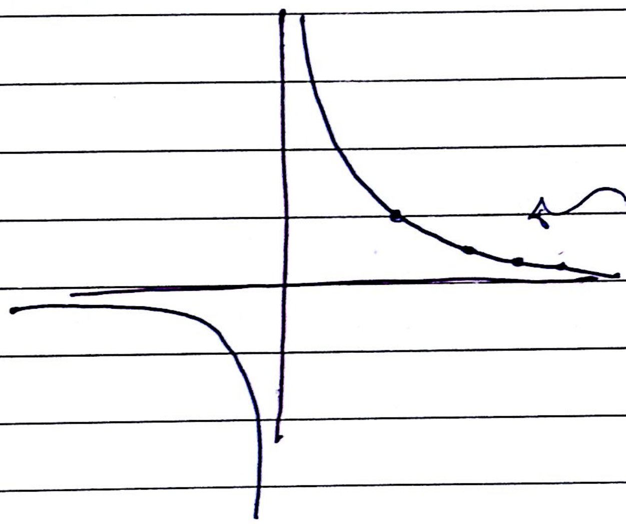


نوعين من النقاط (فوق محور السينات
بواحد وتحت محور السينات بواحد)

ببساطة مناهج

a sequence $\{a_n\}_{n=1}^{\infty}$ can be viewed as points on a function $f(x) = a_x$

Ex given the sequence $\{\frac{1}{n}\}_{n=1}^{\infty}$, then the points of this sequence is a part of the function $f(x) = \frac{1}{x}$.



نقاط المتتالية
تقع على الاقتران #

The limits of the sequence :

Given a sequence $\{a_n\}_{n=1}^{\infty}$, then if whenever n approaches ∞, a_n approaches a real number L, then we say that the limit of this sequence is L, written

$$\lim_{n\to\infty} a_n = L$$

Other wise ~~we say that~~ we say that the limit doesn't exist

Ex find the limit of the following sequences if exist

[1] $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$

$a_n = \frac{1}{n}$

$\lim_{n \to \infty} a_n = 0$

[2] $\left\{ \left(\frac{2}{3}\right)^n \right\}_{n=1}^{\infty}$ $a_n = \left(\frac{2}{3}\right)^n$

$\lim_{n \to \infty} a_n = 0$

In General $a_n = r^n$ $|r| < 1$

then $\Rightarrow \lim_{n \to \infty} a_n = \lim_{n \to \infty} r^n = 0$

when $|r| > 1$

$\lim_{n \to \infty} r^n$ does not exist.

$\boxed{3}$ $\left\{ n\sin\left(\frac{1}{n}\right) \right\}_{n=1}^{\infty}$

$$\lim_{n\to\infty} n\sin\frac{1}{n} = \infty \cdot 0$$

$$= \lim_{n\to\infty} \frac{\sin\left(\frac{1}{n}\right)}{\frac{1}{n}} = \frac{0}{0}$$

كال قاعدة لوبيتال

$$= \lim_{n\to\infty} \frac{\cos\left(\frac{1}{n}\right)\left(-\cancel{\frac{1}{n^2}}\right)}{\cancel{\frac{-1}{n^2}}}$$

$$= \lim_{n\to\infty} \cos\left(\frac{1}{n}\right) = 1$$

method 2 :

$$\lim_{n\to\infty} n\sin\left(\frac{1}{n}\right)$$

$u = \frac{1}{n}$

$n \to \infty$

$u \to 0$

$$\lim_{u\to 0} \frac{1}{u}\sin(u)$$

[4] $1, \sqrt{2}, \sqrt[3]{3}, \sqrt[4]{4}, \sqrt[5]{5}, \ldots$

$a_n = (n)^{\frac{1}{n}}$

$$\lim_{n\to\infty} (n)^{\frac{1}{n}} = \lim_{n\to\infty} e^{\ln(n)^{\frac{1}{n}}}$$

$$= \lim_{n\to\infty} e^{\frac{1}{n}\ln(n)}$$ نستطيع ادخال ال lim للداخل لأنه e متصل.

$$= e^{\lim_{n\to\infty} \frac{\ln(n)}{n}}$$ ثم عد لوبيتال

$$= e^{\lim_{n\to\infty} \frac{\frac{1}{n}}{1}} = e^{0} = 1$$

if $y = f(x)$ is a continous function and $\{a_n\}_{n=1}^{\infty}$ is a sequance, then $\{f(a_n)\}_{n=1}^{\infty}$

then $\lim_{n\to\infty} f(a_n) = f\left(\lim_{n\to\infty} a_n\right)$

$= f(L)$, when $\lim_{n\to\infty} a_n = L$

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Ex $\left\{ \cos^2(\tan^{-1}(n)) \right\}_{n=1}^{\infty}$ $y = \cos^2 x$ and its a continuous function

$= \lim_{n \to \infty} \cos^2(\tan^{-1}(n))$

$= \cos^2 \lim_{n \to \infty} (\tan^{-1}(n))$

$= \cos^2 \frac{\pi}{2} = 0$

Consider the sequence

$\left\{ \left(1 + \frac{a}{n}\right)^n \right\}_{n=1}^{\infty}$, a is a real number

$\lim_{n \to \infty} \left(1 + \frac{a}{n}\right)^n$

$= \lim_{n \to \infty} e^{\ln\left(1 + \frac{a}{n}\right)^n}$

$= e^{\lim_{n \to \infty} n \ln\left(1 + \frac{a}{n}\right)} = e^{\lim_{n \to \infty} \frac{\ln\left(1 + \frac{a}{n}\right)}{\frac{1}{n}}}$

$= e^{\lim_{n \to \infty} \frac{\left(-\frac{a}{n^2}\right) * \frac{1}{1 + \frac{a}{n}}}{-\frac{1}{n^2}}}$

$= e^{\lim_{n \to \infty} \frac{a}{1 + \frac{a}{n}}} = e^{\frac{a}{1}} = e^a$

* $\left\{\left(1+\frac{2}{n}\right)^{n}\right\}_{n=1}^{\infty}$

$$\lim_{n\to\infty}\left(1+\frac{2}{n}\right)^{n} = e^{2}$$

* $\left\{\left(1-\frac{1}{n}\right)^{n}\right\}_{n=1}^{\infty}$ =

$$\lim_{n\to\infty}\left(1+\frac{-1}{n}\right)^{n} = e^{-1}$$

* $\left\{\left(2-\frac{1}{n}\right)^{n}\right\}_{n=}^{\infty}$ ليس الصورة الموضحة

$$\lim_{n\to\infty} 2^{n}\left(1-\frac{1}{2n}\right)^{n} = \infty$$ does not exist

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The Sequeezing Theorem.

Given three sequences $\{a_n\}, \{b_n\}, \{c_n\}$ such that

$$b_n \le a_n \le c_n \qquad \text{for all } n \ge K$$

for some K

and $\lim_{n\to\infty} b_n = L = \lim_{n\to\infty} c_n$ then

$\lim_{n\to\infty} a_n = L$ too.

* Examples:-

[1] $\left\{ \frac{1}{n} \sin(n) \right\}_{n=1}^{\infty}$

$$-\frac{1}{n} \le \frac{1}{n} \le \frac{1}{n} \qquad \text{for all } n = 1, 2, 3, \ldots$$

But $\lim_{n\to\infty} -\frac{1}{n} = 0 = \lim_{n\to\infty} \frac{1}{n}$, so by sequeezing theorem

$$\lim_{n\to\infty} \frac{\sin(n)}{n} = 0$$

^^

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2 $\left\{ \frac{5-3\cos(n)}{n} \right\}_{n=1}^{\infty}$

$\frac{2}{n} \le \frac{5-3\cos(n)}{n} \le \frac{8}{n}$ for all $n = 1, 2, 3, \ldots$

Since $\lim_{n\to\infty} \frac{2}{n} = 0 = \lim_{n\to\infty} \frac{8}{n}$

by sequeezing theorem $\lim_{n\to\infty} \frac{5-3\cos(n)}{n} = 0$

3 $a_n = \frac{n!}{n^n}$

$\frac{n!}{n^n} = \frac{(1)(2)(3)(4)(5)\cdots(n)}{(n)(n)(n)(n)(n)\cdots(n)}$

البرهان

$0 \le \left(\frac{1}{n}\right)\left(\frac{2}{n}\right)\left(\frac{3}{n}\right)\left(\frac{4}{n}\right)\cdots\left(\frac{n}{n}\right) \le \frac{1}{n}$

Since $\lim_{n\to\infty} \frac{1}{n} = 0 = \lim_{n\to\infty} 0$

by sequeezing theorem $\lim_{n\to\infty} \frac{n!}{n^n} = 0$

$\boxed{4}$ $\left\{ a_n = \dfrac{10^n}{n^n} \right\}_{n=1}^{\infty}$

$$0 \le \frac{10^n}{n^n} = \underbrace{\left(\frac{10}{n}\right)\left(\frac{10}{n}\right)\left(\frac{10}{n}\right)\cdots\left(\frac{10}{n}\right)}_{n \text{ times}} \le \frac{10}{n}$$

for all $n \ge 11$

since $\lim\limits_{n \to \infty} 0 = 0 = \lim\limits_{n \to \infty} \dfrac{10}{n}$

by sequeezing theorem $\lim\limits_{n \to \infty} \dfrac{10^n}{n^n} = 0$

مش دائماً بكون صفر !!

Let $y = f(x)$ be a continous function and $\lim\limits_{n \to \infty} a_n = L$ then limits of the sequence $\{f(a_n)\}_{n=1}^{\infty}$ — Remember

$$\lim_{n \to \infty} f(a_n) = f\left(\lim_{n \to \infty} a_n\right) = f(L)$$

Ex Find the limit of the sequence

$$\left\{ \cos^2\left(\frac{3n^2+1}{n^2-1}\right) \right\}_{n=1}^{\infty}$$

قول يا الله ده عليك مش بعيد !!

$$\lim_{n\to\infty} \cos^2\left(\frac{3n^2+1}{n^2-1}\right)$$

$$\cos^2 \lim_{n\to\infty} \frac{3n^2+1}{n^2-1}$$ ⟸ إما بقاعدة لوبيتال او بنقسم على اعلى اس

$$\cos^2 \lim_{n\to\infty} \frac{3+\frac{1}{n^2}}{1-\frac{1}{n^2}}$$

$$\cos^2(3)$$ ⟸ الزاوية بالراديان

Recuirsion sequences:

Consider the sequence

$\sqrt{2}\ ,\ \sqrt{2+\sqrt{2}}\ ,\ \sqrt{2+\sqrt{2+\sqrt{2}}}\ ,\ \sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}\ ,\ \ldots$ (سؤال فاينال)

⟶ $a_1 = \sqrt{2}$

$a_2 = \sqrt{2+\sqrt{2}}$

$a_3 = \sqrt{2+\sqrt{2+\sqrt{2}}}$

- $a_n = \sqrt{2+a_{n-1}}$ $\quad n \geq 2$

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When the n-th term is defind in terms of the previous term, the sequence is called a recursion sequence.

If $\lim_{n\to\infty} a_n = L$ هيا بنا لنجيب L من a_n

$\lim_{n\to\infty} a_{n-1} = L$

So $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \sqrt{2+a_{n-1}}$

$$L = \sqrt{\lim_{n\to\infty} 2+a_{n-1}}$$

$$L = \sqrt{2+L}$$

$$L^2 = 2+L$$

$$L^2 - L - 2 = 0$$

$$(L-2)(L+1) = 0$$

$$L = 2 \,,\, \cancel{-1}$$

لأنه حدود بالموجبة مستحيل ان a_n يكون سالب فنهمله

since $\lim_{n\to\infty} a_n \geq 0$

So $L = 2$ is $\lim_{n\to\infty} a_n = 2$

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Ex $\left\{1+\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\dots+\frac{1}{3^n}\right\}_{n=1}^{\infty}$

$a_1 = 1+\frac{1}{3}$

$a_2 = 1+\frac{1}{3}+\frac{1}{3^2}$

$a_3 = 1+\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}$

$\frac{1}{3}a_n = \frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\dots+\frac{1}{3^n}+\frac{1}{3^{n+1}}$

$a_n - \frac{1}{3}a_n = \frac{1}{\cancel{3}} - \frac{1}{3^{n+1}}$

$\frac{2}{3}a_n = 1 - \frac{1}{3^{n+1}}$

$a_n = \frac{3}{2}\left(1-\frac{1}{3^{n+1}}\right)$

$\lim_{n\to\infty} a_n = \frac{3}{2}(1-0) = \frac{3}{2}$

what's this ..

Increasing and decreasing sequences

* a sequence $\{a_n\}_{n=1}^{\infty}$ is Called :-

1 increasing if

$$a_1 < a_2 < a_3 < a_4 \ldots\ldots$$

2 Decreasing if

$$a_1 > a_2 > a_3 > a_4, \ldots$$

يوجد بعض ~~المتسلسلة~~ المتتاليات
لا تتزايد ولا تتناقص #

* Consider the sequence $\left\{\frac{1}{n}\right\}_{n=1}^{\infty}$

$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$

الحل :- هناك ثلاث طرق :-

Method 1 :-

$f(x) = \frac{1}{x}$ نختبر التزايد والتناقص
(المشتقة الأولى)

$f'(x) = \frac{-1}{x^2} < 0$ on $[1, \infty)$ so $y = f(x) = \frac{1}{x}$ is decreasing on $[1, \infty)$.

Thus $\left\{\frac{1}{n}\right\}_{n=1}^{\infty}$ is decreasing

* Method 2 *

Consider $\frac{a_{n+1}}{a_n} = \frac{\frac{1}{n+1}}{\frac{1}{n}} = \frac{n}{n+1} < 1$

since $n < n+1$

So, $\left\{\frac{1}{n}\right\}_{n=1}^{\infty}$ is decreasing

* Method 3 *

Consider $a_{n+1} - a_n = \frac{1}{n+1} - \frac{1}{n}$

$= \frac{n-(n+1)}{(n+1)n} = \frac{-1}{(n+1)n} < 0$

So $\left\{\frac{1}{n}\right\}_{n=1}^{\infty}$ is decreasing.

Exampels:-

لما يكون في مضروب أسهل طريقة هي القسمة

مثلاً $= 6! = 6 * 5!$

(1) $\left\{\frac{n!}{n^n}\right\}_{n=1}^{\infty}$

Consider

$\frac{a_{n+1}}{a_n} = \frac{\frac{(n+1)!}{(n+1)^n}}{\frac{n!}{(n)^n}} = \frac{\cancel{(n+1)}\,!}{(n+1)^{n+1}} * \frac{n^n}{n!} = \frac{n^n}{(n+1)^n}$

$\left(\frac{n}{n+1}\right)^n = \left(\frac{n+1-1}{n+1}\right)^n = \left(1 - \frac{1}{n+1}\right)^n < 1$

since $n = 1, 2, 3, \ldots$

$0 < 1 - \frac{1}{n+1} < 1$

So the sequence $\left\{ \frac{n!}{n^n} \right\}_{n=1}^{\infty}$ is decreasing

$\boxed{3}$ $\left\{ \tan^{-1} n \right\}_{n\geq 1}^{\infty}$

$f(x) = \tan^{-1} x$

$f'(x) = \frac{1}{1+x^2} > 0$

So, $f(x) = \tan^{-1} x$ is increasing.

H.W $\left\{ \frac{\tan^{-1}(n)}{n^2+1} \right\}_{n=1}^{\infty}$

$\boxed{4}$ $\left\{ \frac{\ln(n)}{n} \right\}_{n=1}^{\infty}$ $\rightsquigarrow$ اسهل اشي الاشتقاق

$f(x) = \frac{\ln(x)}{x} \rightsquigarrow f'(x) = \frac{\frac{x}{x} - \ln x}{x^2} = \frac{1 - \ln x}{x^2} < 0$

on (e, ∞)

So $f(x) = \frac{\ln x}{x}$ is decreasing

on $[e, \infty)$

So $\left\{ \frac{\ln x}{x} \right\}_{n=1}^{\infty}$ is decreasing on $a = 3, 4, \ldots$

Series # المتسلسلات

A series of number is a formal sum of the form in $a_1+a_2+a_3+a_4 \cdots$ where a_1, a_2, a_3, a_4 are real number called the terms of the series

هي حاصل المجموع لحدود المتتالية

Examples

[1] $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots$ مجموع

[2] $1+(-1)+1+(-1)+1+(-1)\cdots$ "

[3] $\sum_{n=2}^{\infty}\frac{1}{n^2-1} = \frac{1}{2^2-1}+\frac{1}{3^2-1}+\frac{1}{4^2-1}+\cdots$

[4] $\sum_{n=2}^{\infty}\ln\left(1-\frac{1}{n}\right) = \ln\left(1-\frac{1}{2}\right)+\ln\left(1-\frac{1}{3}\right)+\cdots$

متتالية المجموع الجزئية لسلسلة

The sequence of Partial Sums of a Series

Given a series $\sum_{n=1}^{\infty} a_n$, then we can construct a new sequence:

$$\{S_n\}_{n=1}^{\infty}$$

$$S_1 = a_1$$
$$S_2 = a_1 + a_2$$
$$S_3 = a_1 + a_2 + a_3$$
$$\vdots$$
$$S_n = a_1 + a_2 + a_3 - - - + a_n$$
$$\vdots$$

* Consider $\lim_{n\to\infty} S_n$

*اذا كانت النهاية موجودة من السلسلة هي Convergent ومجموع = النهاية

if $\lim_{n\to\infty} S_n = L$ then we say that the series is **Convergent** and $\text{Sum} = L$

*اذا كانت النهاية غير موجودة من السلسلة divergent

if $\lim_{n\to\infty} S_n$ doesn't exist then we say that the series is **divergent**

Are the following series Convergent or divergent? if the series is convergent then find it's sum?

$\boxed{1}$ $\sum_{n=1}^{\infty} \frac{1}{2^n} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$

The sequence of partial sum.

$S_1 = \frac{1}{2}$

$S_2 = \frac{1}{2} + \frac{1}{4}$

$S_3 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$

$\vdots$

$S_n = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^n}$

نطرحهم ^^

$\frac{1}{2} S_n = \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \cdots + \frac{1}{2^{n+1}}$

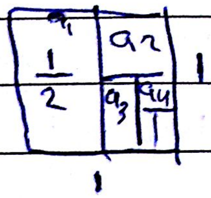
$S_n - \frac{1}{2} S_n = \frac{1}{2} - \frac{1}{2^{n+1}}$

$\frac{1}{2} S_n = \frac{1}{2} - \frac{1}{2^{n+1}}$

$S_n = 2\left(\frac{1}{2} - \frac{1}{2^{n+1}}\right)$ → the closed forms of S_n

$\lim_{n \to \infty} S_n = 1$

So the series of Convergent with sum = 1



مثلا مساحة المربع = مجموع الحدود

$\boxed{2}$ $1+(-1)+1+(-1)+ - -$

$S_1 = 1$

$S_2 = 1 + -1 = 0$

$S_3 = 0 + 1 = 1$

$S_4 = 1 + -1 = 0$

$$S_n = \begin{cases} 1 & \text{if } n = \text{odd} \\ -1 & , n = \text{even} \end{cases}$$

$\lim_{n \to \infty} S_n$ does not exist because the sequence S_n oscillating.

Thus the series is divergent.

$\boxed{3}$ $\displaystyle\sum_{n=2}^{\infty} \ln\left(1-\frac{1}{n}\right)$ سؤال سنوات مهم

$S_1 = \ln\left(1-\frac{1}{2}\right)$

$S_2 = \ln\left(1-\frac{1}{2}\right) + \ln\left(1-\frac{1}{3}\right)$

$S_3 = \ln\left(1-\frac{1}{2}\right) + \ln\left(1-\frac{1}{3}\right) + \ln\left(1-\frac{1}{4}\right)$

$\vdots$

$S_n = \ln\left(1-\frac{1}{2}\right) + \ln\left(1-\frac{1}{3}\right) + \dots + \ln\left(1-\frac{1}{n+1}\right)$

لأنو بدينا عند 2

$a_n = \ln\left(1-\frac{1}{n+1}\right)$

$= \ln\left(\frac{n+1-1}{n+1}\right)$

$= \ln(n) - \ln(n+1)$

$S_1 = \ln(2) - \ln(3)$

$S_2 = \ln(2) - \cancel{\ln(3)} + \cancel{\ln(3)} - \ln(4)$

$= \ln(2) - \ln(4)$

$S_3 = \ln(2) - \cancel{\ln(4)} + \cancel{\ln(4)} - \ln(5)$

$= \ln(2) - \ln(5)$

$\vdots$

$S_n = \ln(2) - \ln(n+2)$

$\lim_{n \to \infty} S_n = -\infty$ does not exist

then the series is divergent

H.W

$$\sum_{n=2}^{\infty} \ln\left(1 - \frac{1}{n^2}\right)$$

* Geometric series

A series of the form

$a + ar + ar^2 + ar^3 + \cdots$

is called a geometric series with first term $= a$ $r =$ constant ratio

$$\frac{a_{n+1}}{a_n} = \frac{ar^n}{ar^{n-1}} = r$$

Ex

1) $\sum_{n=1}^{\infty} \frac{1}{2^n} = \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots$

$a = \frac{1}{2}$ $\frac{a_{n+1}}{a_n} = \frac{\frac{1}{2^{n+1}}}{\frac{1}{2^n}} = \frac{2^n}{2^{n+1}} = \frac{1}{2}$

This is a geometric series with

$a = \frac{1}{2}$ $r = \frac{1}{2}$

2) $5 - \frac{5}{3} + \frac{5}{3^2} - \frac{5}{3^3} + \frac{5}{3^4} + \dots$

$a = 5$, $r = -\frac{1}{3}$ This is ageometric series

3) $1 - 1 + -1 + 1 - 1 \dots$

$a = 1$, $r = -1$ This is ageometric series.

4) $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$

This is not ageometric series because

$a_n = \frac{1}{n}$, so

$\frac{a_{n+1}}{a_n} = \frac{\frac{1}{n+1}}{\frac{1}{n}} = \frac{n}{n+1}$ not aconstant.

* when the geometric series is aconvergent?

Answer

This ageometric series is Convergent when $|r| < 1$ with $\text{sum} = \frac{a}{1-r}$

other wise it is divergent.

Consider the Geometric series $a + ar + ar^2 + ar^{n-1} \ldots$ Find the sequence of partial sums.

$S_1 = a$

$S_2 = a + ar$

$S_3 = a + ar + ar^2$

$\vdots$

$S_n = a + ar + \ldots + ar^{n-1}$

نضرب ب (r)

نعوض

Consider

$rS_n = ar + ar^2 + \ldots ar^n$

$\Rightarrow S_n - rS_n = a + ar + ar^2 + \ldots + ar^{n-1} - (ar + ar^2 + \ldots ar^n)$

$S_n(1-r) = a(1-r^n)$

$$S_n = \frac{a(1-r^n)}{1-r}, \quad r \neq 1$$

$$\lim_{n \to \infty} S_n = \lim_{r \to \infty} \frac{a(1-r^n)}{1-r}, \quad r \neq 1$$

$$\begin{cases} \dfrac{a}{1-r} & \text{when } -1 < r < 1 \\ \text{does not exist} & \text{, other wise} \end{cases}$$

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For $r=1$ we get $a+a+a+\cdots$

This series is divergent because

$\lim_{n\to\infty} S_n = \mp\infty$ (Does not exist)

Example

Are the following series Convergent or divergent? if it is Convergent find the sum.

[1] $\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}+\cdots$

This is ageometric series with $a=\frac{1}{2}$, $r=\frac{1}{2}$

$-1<r<1$, so it is Convergent

with $\text{sum} = \frac{a}{1-r} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1$

[2] $5-\frac{5}{3}+\frac{5}{3^2}-\frac{5}{3^3}+\cdots$

This is ageometric Series with $a=5$

$-1<r=-\frac{1}{3}<1$, The series is Convergent with

$\text{Sum} = \frac{a}{1-r} = \frac{5}{1+\frac{1}{3}} = \frac{15}{4}$

$\boxed{3}$ $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{2^{n+1}}$ نعم هل نستطيع كتابته على الشكل النموذجي ar^{n-1}

$$= \sum_{n=1}^{\infty} \frac{3 \cdot 3^{n-1}}{2^2 (2^{n-1})}$$

$$= \sum_{n=1}^{\infty} \frac{3}{4} \left(\frac{3^{n-1}}{2^{n-1}} \right)$$

This is a geometric series with $a = \frac{3}{4}$

$r = \frac{3}{2} > 1$

So the series is divergent.

الكسر العشري الدوري

EX Represent the repeating decimal

$0.\overline{21}$ as a proper fraction

$$0.\overline{21} = \frac{21}{100} + \frac{21}{100^2} + \frac{21}{100^3} + \cdots$$

$0.21212121\ldots$

This is a geometric series with $a = \frac{21}{100}$

$r = \frac{1}{100}$

So it is convergent with

$$\text{Sum} = \frac{\frac{21}{100}}{\frac{100}{100} - \frac{1}{100}} = \frac{\frac{21}{100}}{\frac{99}{100}} = \frac{21}{99}$$

* $2.\overline{35} = 2 + .\overline{35}$

$$= 2 + \frac{\frac{35}{100}}{1 - \frac{1}{100}}$$

$$= 2 + \frac{35}{99}$$

$$\frac{198 + 35}{99} = \frac{233}{99} \rightarrow \underline{\text{sum}}$$ ^^

Telescoping Series

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \frac{1}{2} + \frac{1}{2(3)} + \frac{1}{(3)(4)} + \dots$$

(not geometric) لهذا نعود للأساس

$S_1 = \frac{1}{2}$

$S_2 = \frac{1}{2} + \frac{1}{2(3)}$

$S_3 = \frac{1}{2} + \frac{1}{2(3)} + \frac{1}{3(4)}$

$\|$

$S_1 = 1 - \frac{1}{2}$

$S_2 = 1 - \cancel{\frac{1}{2}} + \cancel{\frac{1}{2}} - \frac{1}{3}$

$= 1 - \frac{1}{3}$

$S_3 = 1 - \cancel{\frac{1}{3}} + \cancel{\frac{1}{3}} - \frac{1}{4}$

$= 1 - \frac{1}{4}$

$S_n = 1 - \frac{1}{n+1} \rightarrow$

$a_n = \frac{1}{n(n+1)}$ ⇐ نفكر بـ partial fraction

$= \frac{A}{n} + \frac{B}{n+1}$

$= \frac{1}{n} - \frac{1}{n+1}$

* سميناه telescoping لأنه بنجمع الي طوله الفرق هو بالحل - كله الأخير (الجمع)

$$\lim_{n \to \infty} S_n = 1$$

the series is Convergent with sum $= 1$

$$\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$$

$$a_n = \frac{1}{n(n+2)} = \frac{A}{n} + \frac{B}{n+2}$$

$$= \frac{\frac{1}{2}}{n} - \frac{\frac{1}{2}}{n+2}$$

$$= \frac{1}{2}\left[\frac{1}{n} - \frac{1}{n+2}\right)$$

$$S_1 = \frac{1}{2}\left[1 - \frac{1}{3}\right)$$

$$S_2 = \frac{1}{2}\left[1 - \frac{1}{3}\right) + \frac{1}{2}\left[\frac{1}{2} - \frac{1}{4}\right]$$

$$S_3 = \frac{1}{2}\left[1 - \cancel{\frac{1}{3}}\right] + \frac{1}{2}\left[\frac{1}{2} - \frac{1}{4}\right] + \frac{1}{2}\left[\cancel{\frac{1}{3}} - \frac{1}{5}\right]$$

$$= \frac{1}{2}\left[1 + \frac{1}{2} - \frac{1}{4} - \frac{1}{5}\right]$$

$$S_4 = \frac{1}{2}\left[1 + \frac{1}{2} - \cancel{\frac{1}{4}} - \frac{1}{5}\right] + \frac{1}{2}\left[\cancel{\frac{1}{4}} - \frac{1}{6}\right]$$

$$= \frac{1}{2}\left[1 + \frac{1}{2} - \frac{1}{5} - \frac{1}{6}\right]$$

$\vdots$

$$S_n = \frac{1}{2}\left[1 + \frac{1}{2} - \frac{1}{n+1} - \frac{1}{n+2}\right]$$

$$\lim_{n \to \infty} S_n = \left[1 + \frac{1}{2} - 0 - 0\right] \frac{1}{2}$$

$$= \frac{1}{2}\left(\frac{3}{2}\right)$$

$$= \frac{3}{4}$$

So, the series is Convergent with sum $= \frac{3}{4}$

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* Given two series $\sum_{n=1}^{\infty} a_n$, $\sum_{n=1}^{\infty} b_n$ إلى التيسير ^^

then ① $\sum_{n=1}^{\infty} a_n + \sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} a_n + b_n$

② $K \sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} K a_n$

H.W

$\sum_{n=3}^{\infty} \frac{1}{n^2-4} =$

* Remark

① $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are convergent with sums L

and M respectively (على الترتيب), then the sum

$\sum_{n=1}^{\infty} a_n + \sum_{n=1}^{\infty} b_n$ is convergent with sum

L+M also $K \sum_{n=1}^{\infty} a_n$ is convergent with sum

KL

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Find the sum (if exist)

$$\sum_{n=1}^{\infty}\left(\frac{2^n-5\times 3^{n+1}}{6^{n-1}}\right)$$

* الحل :-

① بنحدد هل هي Convergent أو Div

② لاحقاً ... !!

الفكرة الأولى :-

* هاي بحالة اذا التسلسل Convergent

$$=\sum_{n=1}^{\infty}\frac{2^n}{6^{n-1}}-\frac{5\times 3^{n+1}}{6^{n-1}}$$

$$=\sum_{n=1}^{\infty}\frac{2^n}{6^{n-1}}-5\sum_{n=1}^{\infty}\frac{3^{n+1}}{6^{n-1}}$$

$$=\sum_{n=1}^{\infty}2\left(\frac{2}{6}\right)^{n-1}-5\sum_{n=1}^{\infty}9\left(\frac{3}{6}\right)^{n-1}$$

* $\sum_{n=1}^{\infty}2\left(\frac{2}{6}\right)^{n-1}$ $\approx$ geometric with $a=2$, $r=\frac{1}{3}$

So, it is Convergent with $\text{sum}=\frac{2}{1-\frac{1}{3}}$

$=3$

* $\sum_{n=1}^{\infty}9\left(\frac{1}{2}\right)^{n-1}$ $\approx$ geometric with $a=9$, $r=\frac{1}{2}$

So, it is Convergent with

$\text{Sum}=\frac{9}{1-\frac{1}{2}}=18$

So, the series is Convergent with $\text{Sum}=3-5(18)$

$=-87$

* Remark :-

الفكرة الثانية ::

If $\sum_{n=1}^{\infty} a_n$ is Convergent and $\sum_{n=1}^{\infty} b_n$ is divergent then the sum is divergent and $K \sum_{n=1}^{\infty} b_n$ is divergent $\Rightarrow K \neq 0$

مع اشتراط K لازم لا تكون صفر

* ملاحظة :-

لو كانوا المتسلسلتين لا يبدؤو من نفس الحد مثلاً :-

$$\sum_{n=3}^{\infty} a_n \quad , \quad \sum_{n=1}^{\infty} b_n$$

نأخذ اول حدين لوحدها و نجعل الحدود متناظرة وجمع .

* Remark :-

If both series are divergent , then the sum might be Convergent or divergent.

* 17/4/2014 *

Divergence Test

Given a series $\sum_{n=1}^{\infty} a_n$

If this series is Convergent, then

$$\lim_{n \to \infty} S_n = L$$

, where S_n: the sequence of partial sums.

$$S_n - S_{n-1} = a_n$$

so

$$\lim_{n \to \infty} S_n - S_{n-1} = \lim_{n \to \infty} a_n$$

$$L - L = 0$$

* اذا كانت Convergent series من النهاية للحد النوني = صفر

* Divergence test.

For a series $\sum_{n=1}^{\infty} a_n$,

if $\lim_{n \to \infty} a_n \neq 0$, then the series is divergent

Examples.

*اذا كانت $\lim$ = صفر

هناك احتمالين: ① اما Div

② او Con

صحيح طبعا يابني

① $\sum_{n=1}^{\infty} n^{\frac{1}{n}}$

$a_n = n^{\frac{1}{n}}$

$\lim_{n \to \infty} a_n = 1$

by the divergence test, the series is divergent

② $\sum_{n=2}^{\infty} \left(1 - \frac{1}{n}\right)^n$

$a_n = \left(1 - \frac{1}{n}\right)^n$

$\lim_{n \to \infty} a_n = e^{-1} \neq 0$

By the divergence test, the series is divergent

③ $\sum_{n=1}^{\infty} \frac{(-1)^n n}{n+1}$

$a_n = \frac{(-1)^n n}{n+1}$

$\lim_{n \to \infty} a_n$ does not exist, because $\lim_{n \to \infty} \frac{n}{1+n} = 1$,

So, the odd terms approach -1, by divergence test the series is divergent

while the even term approach

The Integral test

Given a series $\sum_{n=1}^{\infty} a_n$, $a_n \geq 0$

مشان اكل n : x

If $f(x) = a_x$ is decreasing continous on

$[c, \infty]$ fore some $c \geq 1$, then the series and the improper integral

$\int_c^{\infty} f(x)\,dx$ either both converge or both

diverge

Ex $\sum_{n=1}^{\infty} \frac{1}{n}$

$a_n = \frac{1}{n} > 0$

- $f_x = \frac{1}{x}$ $\rightarrow f'(x) = -\frac{1}{x^2} < 0$ on $[1, \infty)$

So it is decreasing continous on $[1, \infty)$

Consider $\int_1^{\infty} \frac{1}{x}\,dx \rightarrow \lim_{b \to \infty} \int_1^{b} \frac{1}{x}\,dx = \lim_{b \to \infty} \ln|x| \Big]_1^b$

$= \lim_{b \to \infty} \ln|b| - \ln 1$

$= \infty$

So divengent

So the series is divergent by integral test

Five App

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$\Rightarrow$ This is called the Harmonic series.

اقفت عيناي شوقاً

$\boxed{2}$ $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$

$$a_n = \frac{1}{n^2} > 0$$

$$f(x) = \frac{1}{x^2} = x^{-2} \quad , \quad f'(x) = -2x^{-3}$$

$$= -\frac{2}{x^3} < 0$$

So, it is deacresing on $[1,\infty)$

Consider $\displaystyle\int_1^{\infty} \frac{1}{x^2} = \lim_{b \to \infty} \int_1^{b} \frac{1}{x^2} = \lim_{b \to \infty} -\frac{1}{x}\Big]_1^{b}$

$$= \lim_{b \to \infty} -\frac{1}{b} + 1$$

$$= 1$$

The improper integral is $[1,\infty)$ Convergent, By the integral test on the series is Convergent

p-series.

A series of the form

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \quad , \quad p > 0$$

عدد حقيقي موجب

is called p-series.

① $\sum_{n=1}^{\infty} \frac{1}{n}$, is a p-series with $p=1$

② $\sum_{n=1}^{\infty} \frac{1}{n^2}$, is 〃 〃 〃 $p=2$

③ $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$, 〃 〃 〃 〃 $p=\frac{1}{2}$

p-series.

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \quad , p > 0$$

using the integral test, one can show that

① the p-series is Convergent when $p > 1$ ^‿^

② " " " divergent when $0 < p \leq 1$ ‼︎

Ex

[1] $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ ⇝ this is a p-series with

$0 < p = \frac{1}{2} < 1$. so, it is divergent.

2] $\sum_{n=2}^{\infty} \frac{5}{n^{\frac{3}{2}}}$ * طرح او اضافة عدد من الحدود لايؤثر على (div. oR conv.)

$\sum_{n=2}^{\infty} \frac{1}{n^{\frac{3}{2}}}$ is a p-series with $p = \frac{3}{2} > 1$

Hence it is convergent, so

$\sum_{n=2}^{\infty} \frac{5}{n^{\frac{3}{2}}}$ is Convergent.

Comparison test

Given a series $\sum_{n=1}^{\infty} a_n$, $a_n \geq 0$

then,

[1] if there exist a series $\sum_{n=1}^{\infty} b_n$, $b_n \geq 0$

with $a_n \leq b_n$ for all $n \geq k$ for some k, and

b_n is Convergent, then a_n is convergent too.

[2] if there exist a series $\sum_{n=1}^{\infty} b_n$, $b_n \geq 0$ and

$b_n \leq a_n$ for all $n \geq k$, for some positive

integer k , ~~then~~ which is divergent , then a_n

is divergent too.

بدو يعطينا أمثلة تطبيقية على هذا الكلام

Examples

(1) $$\sum_{n=1}^{\infty} \frac{1}{n^4+3n+10}$$

سؤال حالو o.O

هل هو دائماً أكبر من صفر ؟؟

$$a_n = \frac{1}{n^4+3n+10} > 0$$

$$a_n = \frac{1}{n^4+3n+10} \leq \frac{1}{n^4}$$

since $3n+10 > 0$

ولازم نثبتها بتصغير المقام

But $\sum_{n=1}^{\infty} \frac{1}{n^4}$ is a p-series $p=4>0$, so it is convergent.

By Comparison test the series $\sum_{n=1}^{\infty} a_n$ is convergent

لازم يكون عندك

احساس ::

:: Hunch ::

$\boxed{2}$ $\displaystyle\sum_{n=5}^{\infty} \frac{1}{\sqrt{n}-2}$

$a_n = \dfrac{1}{\sqrt{n}-2} > \dfrac{1}{\sqrt{n}}$ for all $n \geq 5$.

$\cancel{\text{But } \frac{1}{\sqrt{n}} \to 0}$

$\displaystyle\sum_{n=5}^{\infty} a_n = \sum_{n=5}^{\infty} \frac{1}{\sqrt{n}}$ is ap-series will

مش عارفة معناها ؟؟

$0 < p = \frac{1}{2} \leq 1$, hence it is divergent

So by Comparison test, the series is divergent.

$\boxed{3}$ $\displaystyle\sum_{n=1}^{\infty} \frac{\sin^2(n)}{n^3}$

الأسئلة لا تنتهي، يجب أن تتدرب !!
(د. حسن العزة)

$a_n = \dfrac{\sin^2(n)}{n^3} \geq 0$

$a_n = \dfrac{\sin^2(n)}{n^3} \leq \dfrac{1}{n^3}$, since $\sin^2(n) \leq 1$

But $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3}$ is ap-series with $p = 3 > 1$, So it is convergent. By Comparison test, the series $\displaystyle\sum_{n=1}^{\infty} \frac{\sin^2(n)}{n^3}$ is convergent.

مش مبني

4) $\sum_{n=1}^{\infty} e^{-n^2}$

$a_n = e^{-n^2} > 0$

$a_n = e^{-n^2} = \frac{1}{e^{n^2}} \leq \frac{1}{e^n}$ since

$n^2 \geq n$, so $-n \geq e^{-n^2}$

and so $e^{-n} \geq e^{-n^2}$

But $\sum_{n=1}^{\infty} \frac{1}{e^n}$ is Convergent.

because it is ageometric series

with $-1 < r = \frac{1}{e} < 1$.

By Comparison test the series $\sum_{n=1}^{\infty} e^{-n^2}$

is Convergent

Limit Comparison test

Given a series $\sum_{n=1}^{\infty} a_n$, $a_n > 0$

If there exist a series $\sum_{n=1}^{\infty} b_n$, $b_n > 0$

with $\lim_{n\to\infty} \frac{a_n}{b_n} = L \neq 0$, then either

both Convergent or both divergent.

Ex

~~1~~

1 $\sum \sin(\frac{1}{n})$

$a_n = \sin(\frac{1}{n})$

Consider $b_n = \frac{1}{n} > 0$

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\sin(\frac{1}{n})}{\frac{1}{n}} = 1$ $\leadsto$ by L-Hopital rule.

$\sum_{n=1}^{\infty} \frac{1}{n}$ is a p-series with $p = 1$ so it is divergent. By limit Comparison test, the series $\sum_{n=1}^{\infty} \sin(\frac{1}{n})$ is divergent.

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$$\sum_{n=1}^{\infty} \frac{n^2+3n-5}{n^{5/2}-5n^2+10}$$

أسهل طريقة لحل مثل هذا النوع من الأسئلة

• ناخذ $b_n =$ اعلى قوة من البسط / اعلى قوة من المقام

$$b_n = \frac{n^2}{n^{5/2}} = \frac{1}{n^{1/2}} > 0$$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n^2+3n-5}{n^{5/2}-5n^2+10} \div \frac{1}{n^{\frac{1}{2}}}$$

$$= \lim_{n\to\infty} \frac{n^{5/2}+3n^{3/2}-5n^{1/2}}{n^{5/2}-5n^2+10}$$

نقسم البسط والمقام على أعلى أس

$$= \lim_{n\to\infty} \frac{1+\frac{3}{n}-\frac{5}{n^2}}{1-\frac{5}{n^{1/2}}+\frac{10}{n^{5/2}}} = 1 \neq 0$$

$\sum_{n=1}^{\infty} \frac{1}{n^{\frac{1}{2}}}$ is a p-series with $0 < p = \frac{1}{2} < 1$

so it is divergent. by limit comparison test the series $\sum_{n=1}^{\infty} a_n$ is divergent.

الدكتور انجبت بحاد لسؤال :

$[2]$ $\sum_{n=1}^{\infty} \frac{4^n}{4^n+3^n}$ ans {past paper}

$a_n = \frac{4^n}{4^n+3^n}$ ans 4^n مقسوم على

$= \frac{1}{1+\left(\frac{3}{4}\right)^n}$ ans بالقسمة على البسط :

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{1+\left(\frac{3}{4}\right)^n} = 1$

By the divergence test, the series is divergent.

Other solution.

$a_n = \frac{4^n}{4^n+3^n} \geq \frac{1}{2}$

$\sum_{n=1}^{\infty} \frac{1}{2}$ is divergent By Comparison test,

the series $\sum_{n=1}^{\infty} \frac{4^n}{4^n+3^n}$ is div.

Ratio Test

أسهل طريقة وأكثر طريقة استخداماً
(على ذمّة الدكتورة)

Given a series $\sum_{n=1}^{\infty} a_n$, $a_n > 0$

$\lim_{n \to \infty} \frac{a_{n+1}}{a_n} = L$, then i.e.

[1] If $L < 1$ the series is Convergent.

[2] If $L > 1$ the series is divergent.

[3] If $L = 1$ the test fails.

Examples

[1] $\sum_{n=1}^{\infty} \frac{n!}{n^n}$

لما تشوف مضروب فهاي أحسن طريقة
لكل

$$\lim_{n \to \infty} \frac{a_{n+1}}{a_n} = \lim_{n \to \infty} \frac{(n+1)!}{(n+1)^{n+1}} \div \frac{n!}{n^n}$$

$$= \lim_{n \to \infty} \frac{(n+1)!}{(n+1)^{n+1}} \times \frac{n^n}{n!}$$

$$= \lim \frac{n^n}{(n+1)^n} = \lim \left(\frac{n}{n+1}\right)^n = \lim_{n \to \infty} \left(\frac{1}{1+\frac{1}{n}}\right)^n$$

$$= \lim_{n \to \infty} \frac{1}{\left(1+\frac{1}{n}\right)^n}$$

$$= \frac{1}{e} < 1$$

, So by ratio test the series is Convergent.

* $\sum_{n=1}^{\infty} \frac{3^n}{n+1}$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{3^{n+1}}{n+2} \div \frac{3^n}{n+1}$$

$$= \lim_{n\to\infty} \frac{3^{n+1}}{3^n} \left(\frac{n+1}{3^n}\right)$$

$$= (3)(1) \quad = 3 > 1$$

By ratio test, the series is divergent.

* $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{3^n}{n+1} \quad = \frac{\infty}{\infty}$ ~~(other way)~~

على لوبيتال

$$\lim_{n\to\infty} \frac{3^n \ln 3}{1} = \infty \quad \text{does not exist}$$

By divergence test the series

is divergent

* $\sum_{n=1}^{\infty} \frac{1}{n^2}$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim \frac{\frac{1}{(n+1)^2}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{n^2}{(n+1)^2}$$

$$= \lim_{n\to\infty} \frac{n^2}{n^2+2n+1}$$

$$= \lim_{n\to\infty} \frac{1}{1+\frac{2}{n}+\frac{1}{n^2}} = 1$$

So, the ratio test is fail.

So, we have to find another method to solved, in fact, this series is a p-series with $p=2>1$, hence it is convergent.

The Root series

Given a series $\sum_{n=1}^{\infty} a_n$, $a_n \geq 0$.

If $\lim_{n\to\infty} (a_n)^{\frac{1}{n}} = L$, then:-

1. when $L<1$, the series is convergent.
2. when $L>1$, ~~the series~~ or $L=\infty$ the series is divergent.
3. when $L=1$, the ~~series~~ test fails.

Example

$$\sum_{n=2}^{\infty} \left(1-\frac{2}{n}\right)^{n^2}$$

a) $\lim_{n\to\infty} (a_n)^{\frac{1}{n}} = \lim_{n\to\infty} \left(\left(1-\frac{2}{n}\right)^{n^2}\right)^{\frac{1}{n}} = \lim_{n\to\infty} \left(1-\frac{2}{n}\right)^{n} = e^{-2} < 1$

So, by root test the series is convergent.

b) what is the $\lim_{n\to\infty} a_n$??

$$\lim_{n\to\infty} a_n = 0$$

Because $\lim_{n\to\infty} a_n$ is convergent as above

$\boxed{2}$ $\sum \left(2 - \frac{1}{n}\right)^n$

$\lim_{n \to \infty} a_n^{\frac{1}{n}}$

$= \lim_{n \to \infty} \left(\left(2 - \frac{1}{n}\right)^n\right)^{\frac{1}{n}} = \lim_{n \to \infty} 2 - \frac{1}{n} = 2 > 1$

By root test, the series is divergent.

$\lim_{n \to \infty} a_n = \lim_{n \to \infty} \left(2 - \frac{1}{n}\right)^n = \lim_{n \to \infty} 2^n \left(1 - \frac{1}{2}n\right)^n$

$= \infty$ $\to$ does not exist

حل امتحان سابق!!

يا رب

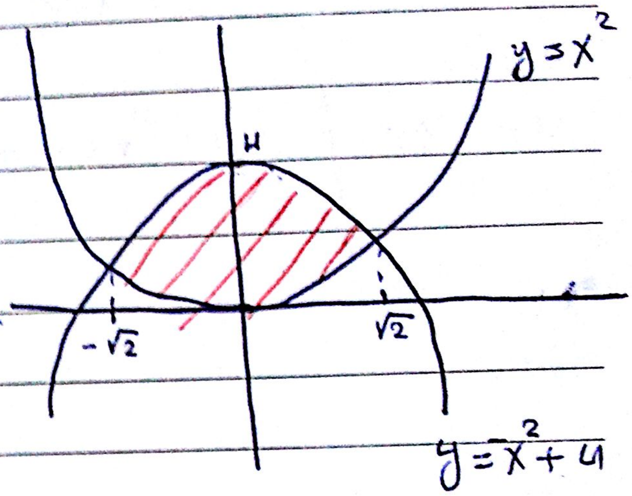
$\boxed{1}$ Find the voluem of the solid who's base is the shaded region and each cross section perpendicular to the x-axis is a semicircle

$x^2 = -x^2 + 4$

$2x^2 = 4$

$x^2 = 2$

$x = \mp\sqrt{2}$



$$A(x) = \frac{\pi}{2}\left(\frac{-x^2+4-x^2}{2}\right)^2$$

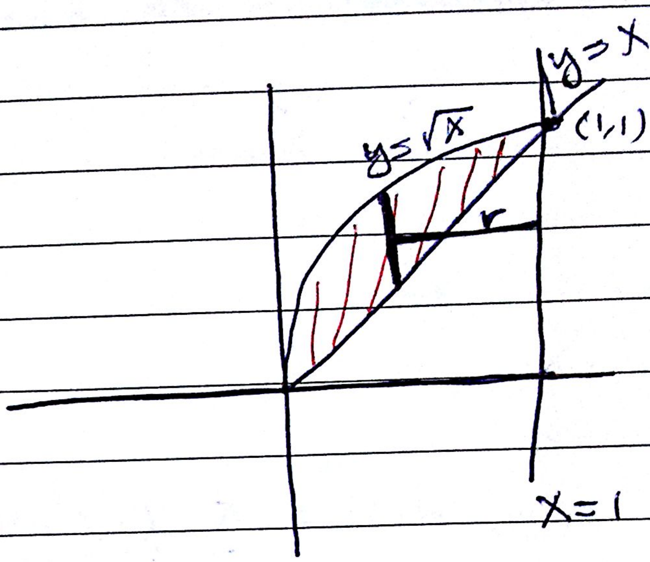
$$= \frac{\pi}{8}\left(4-2x^2\right)^2$$

$$V = \int_{-\sqrt{2}}^{\sqrt{2}} \frac{\pi}{8}\left(4-2x^2\right)^2$$

2) Use the shell method to find the volum of the solid generated by rotating the shaded region about $x=1$

$r = 1-x$

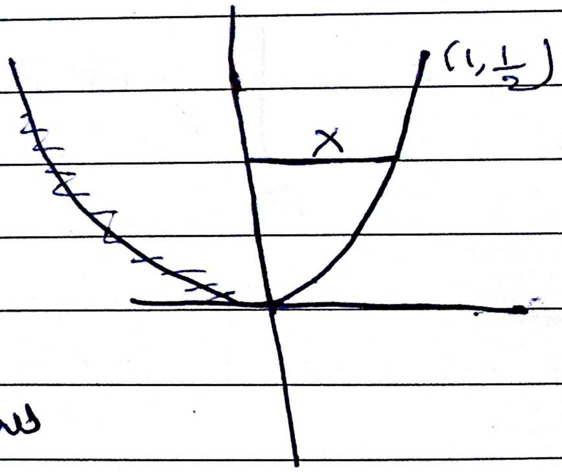
$$V = 2\pi \int_0^1 (1-x)(\sqrt{x}-x)\,dx$$



^^ جيد

3) Find the area of the surface generated by rotating the curve $y=\frac{1}{2}x^2$, $0 \leq x \leq 1$ about the y-axis.

$$S = \int_0^1 2\pi x\sqrt{1+x^2}\;dx$$



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$\boxed{4}$ Find the limit of the sequence.

$$\left\{\left[\sin^2\left(1+\frac{3}{n}\right)^{-n}\right]\right\}_{n=1}^{\infty}$$

$$\lim_{n\to\infty} \sin^2\left(1+\frac{3}{n}\right)^{-n}$$

$$= \sin^2 \lim\left(\frac{1}{\left(1+\frac{3}{n}\right)^n}\right)$$

$$= \sin^2\left(e^{-3}\right)$$

$\boxed{5}$ Find the sum (if exists)

$$\sum \frac{1}{n(n+2)} + \frac{3^{n-1}}{4^n}$$

Consider $\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$

$$a_n = \frac{\frac{1}{2}}{n} - \frac{\frac{1}{2}}{n+2}$$

$$= \frac{1}{2}\left(\frac{1}{n} - \frac{1}{n+2}\right)$$

$$S_1 = \frac{1}{2}\left(1-\frac{1}{3}\right)$$

$$S_2 = \frac{1}{2}\left(1-\frac{1}{3}\right) + \frac{1}{2}\left(\frac{1}{2}-\frac{1}{4}\right)$$

$$S_3 = \frac{1}{2}\left(1-\cancel{\frac{1}{3}}\right) + \frac{1}{2}\left(\frac{1}{2}-\frac{1}{4}\right) + \frac{1}{2}\left(\cancel{\frac{1}{3}}-\frac{1}{5}\right)$$

$$\vdots$$

$$S_n = \cancel{\frac{1}{2}\left(1-\frac{1}{3}\right) + \frac{1}{2}\left(\frac{1}{2}-\frac{1}{4}\right) + \frac{1}{2}\left(\frac{1}{3}-\frac{1}{5}\right)}$$

$$= \frac{1}{2}\left[1+\frac{1}{2}-\frac{1}{n+1}-\frac{1}{n+2}\right]$$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{1}{2}\left[1+\frac{1}{2}-\frac{1}{n+1}-\frac{1}{n+2}\right]$$

$$= \frac{1}{2}\left[1+\frac{1}{2}-0-0\right]$$

$$= \frac{1}{2} * \frac{3}{2}$$

$$= \frac{3}{4}$$

$$\sum_{n=1}^{\infty} \frac{3^{n-1}}{4^n} = \sum_{n=1}^{\infty} \frac{1}{4}\left(\frac{3}{4}\right)^{n-1}$$

This is ageometric with $a = \frac{1}{4}$ $r = \frac{3}{4}$

So, it is Conv. with sum $= \dfrac{\frac{1}{4}}{1-\frac{3}{4}} = 1$

$$\text{Sum} = \frac{3}{4} + 1 = \frac{7}{4} \ \#$$

6 Test for Convergence. جاي بالامتحان يكون اكثر من فرع. مهم

a) $\sum_{n=2}^{\infty} \frac{1}{n(\ln(n))^2}$

By Integral test

$a_n = \frac{1}{n(\ln(n))^2} > 0$

$f(x) = \frac{1}{x(\ln(x))^2}$, this is decreasing Continous on $[2, \infty)$

نشتق عشان نشوف التناقص.

$$f'(x) = \frac{1}{(x(\ln(x))^2)^2} \left(\cancel{x} \cdot 2 \cdot \ln(x) \cdot \frac{1}{\cancel{x}} - (\ln x)^2\right)$$

$$= \frac{2\ln x - (\ln x)^2}{(x \cdot (\ln x)^2)^2} < 0 \quad \text{on } (e, \infty)$$

$$\int_2^{\infty} \frac{dx}{x(\ln x)^2} = \lim_{b \to \infty} \int_2^{b} \frac{dx}{x(\ln x)^2}$$

$$= \lim_{b \to \infty} -\frac{1}{\ln x}\Big]_2^b$$

$$= \lim_{b \to \infty} \frac{-1}{\ln(b)} + \frac{1}{\ln 2}$$

$$= \frac{1}{\ln(2)} \quad \text{Conv.}$$

So, by integral test, the series is conv.

b) $\sum_{n=1}^{\infty} \frac{2}{3^n+5}$

نستخدم Comparison:

$a_n = \frac{2}{3^n+5} > 0$

$\frac{2}{3^n+5} < \frac{2}{3^n}$

$\sum_{n=1}^{\infty} \frac{2}{3^n}$ is a geometric ~~test~~ with $-1 < r = \frac{1}{3} < 1$

hence it is Conv.

By Comparison test, the series $\sum_{n=1}^{\infty} a_n$ is C

(another solution)

$b_n = \frac{1}{3^n} > 0$

$\lim_{n \to \infty} \frac{a_n}{b_n} = \lim \frac{2}{3^n+5} \div \frac{1}{3^n}$

$= \lim_{n \to \infty} \frac{2 \cdot 3^n}{3^n+5} = \lim_{n \to \infty} \frac{2}{1+\frac{5}{3^n}} = \frac{2}{1+0} = 2 \neq 0$

But $\sum_{n=1}^{\infty} \frac{1}{3^n}$ is Conv., It is geometric

So, by limit Comparison test the $\sum a_n$ is Conv.

d) $\sum_{n=1}^{\infty} \tan^{-1}(n)$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \tan^{-1}(n)$$

$$= \frac{\pi}{2}$$

By divergence test it is divergent

إدعولي !!

الامتحان كان 10:30

بالتوفيق شهد